Modelisation de la mobilite intra-urbaine a l'aide de mesures hyperfractales dans le plan.

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INRIA

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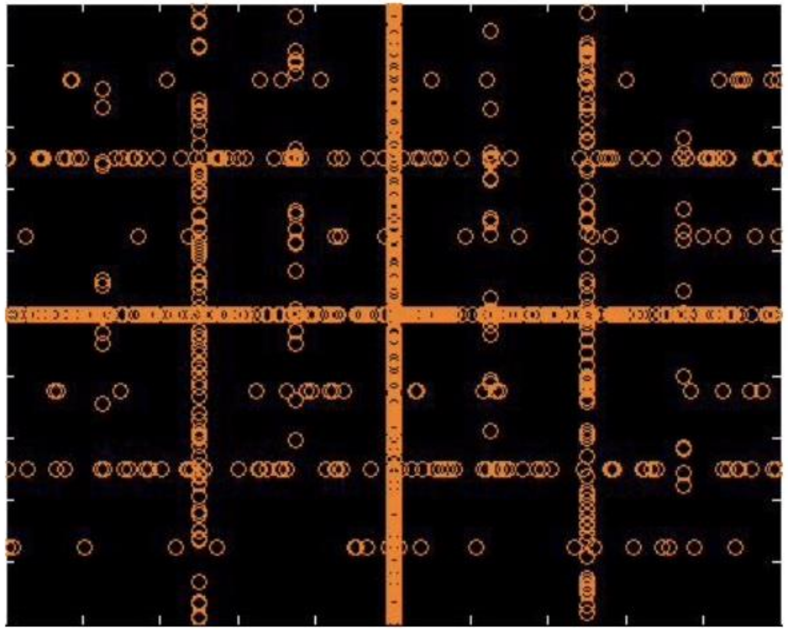
Et du PEPR MobiDec

¼ de siecle pour ¼ de plan. Marseille 15-17 avril 2025

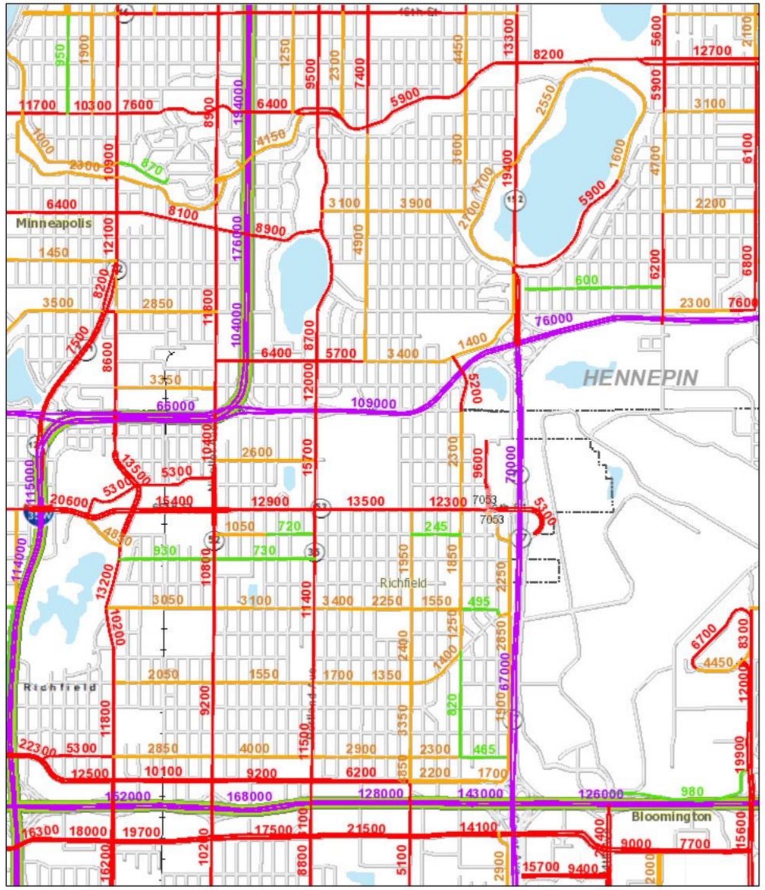
Flyover in cities

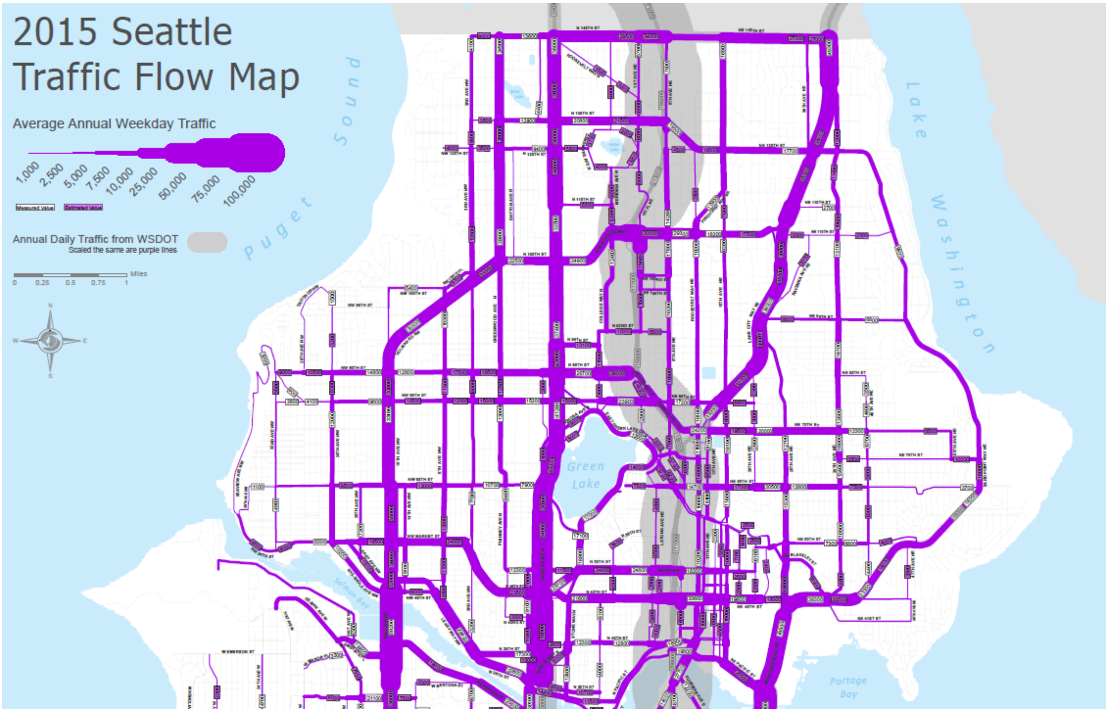
- actual and model



Car Street density

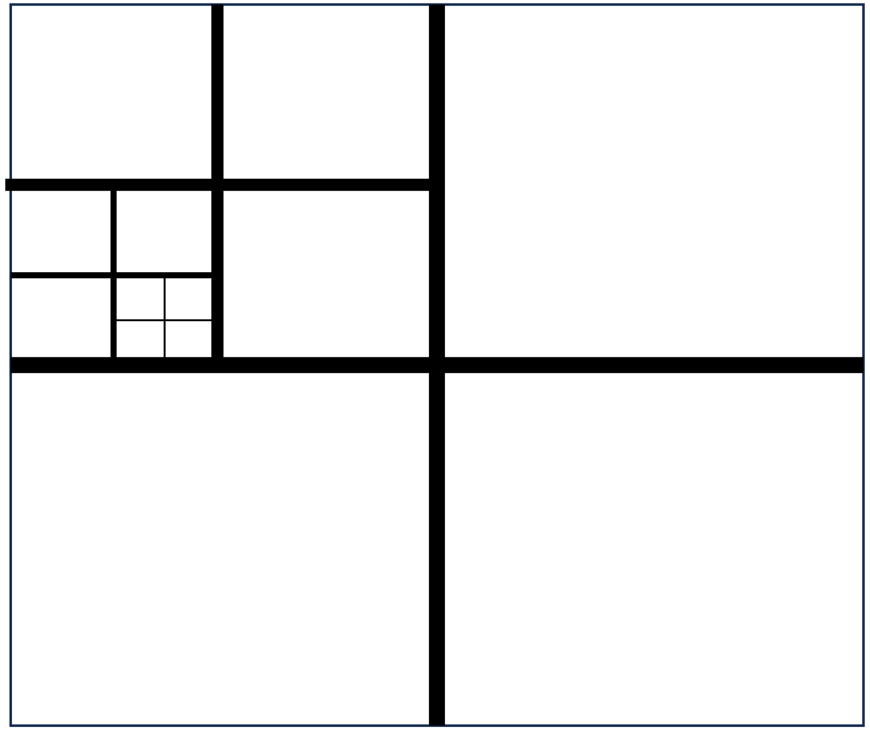
- Actual density map Mineapolis, Seattle

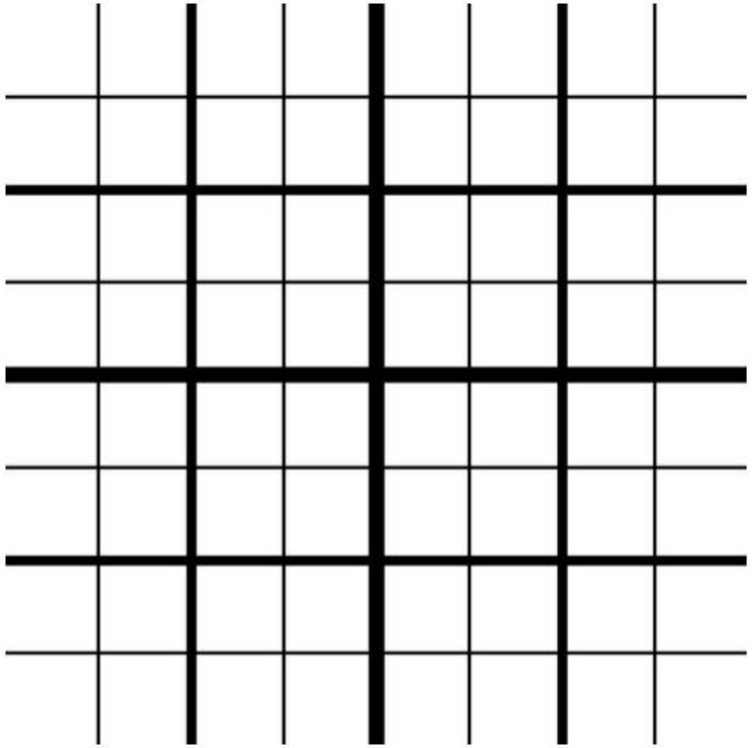




Hyperfractal construction

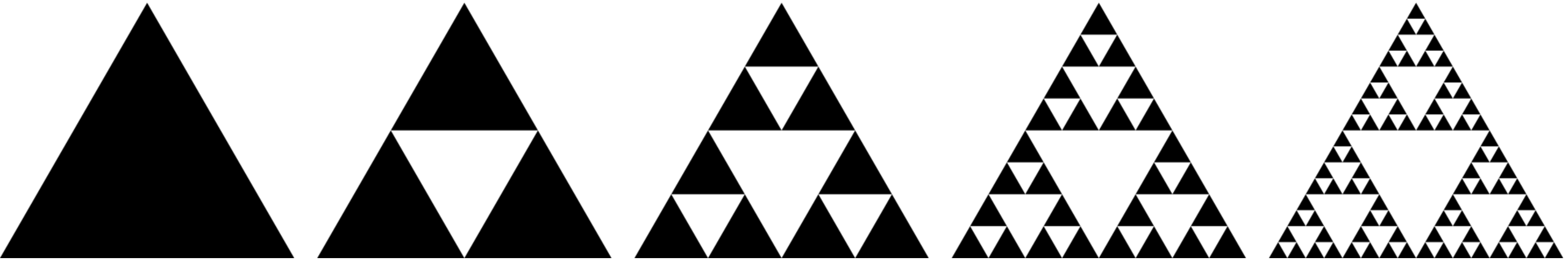
- One parameter $0<p<1$, $q=1-p$. A unit square and a density n



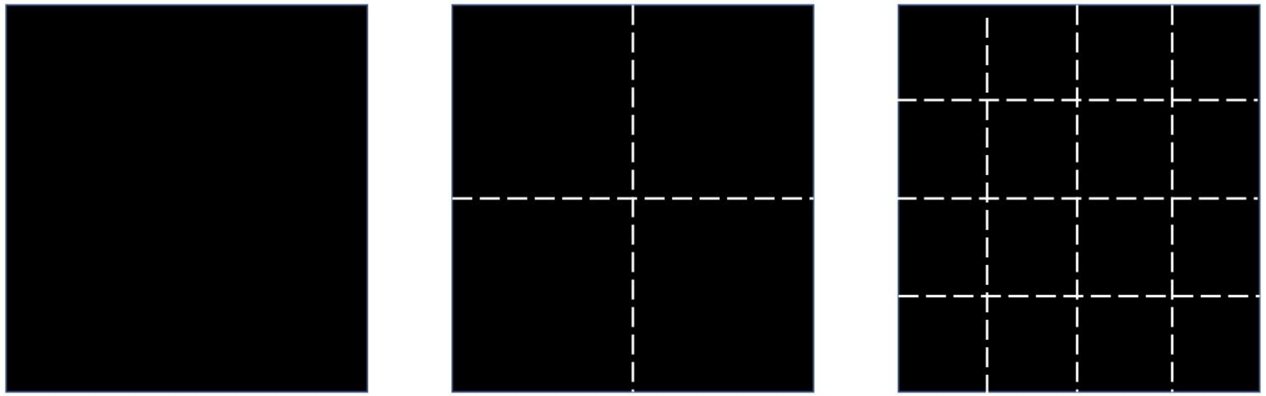


Fractal dimension

- The Serpinski triangle

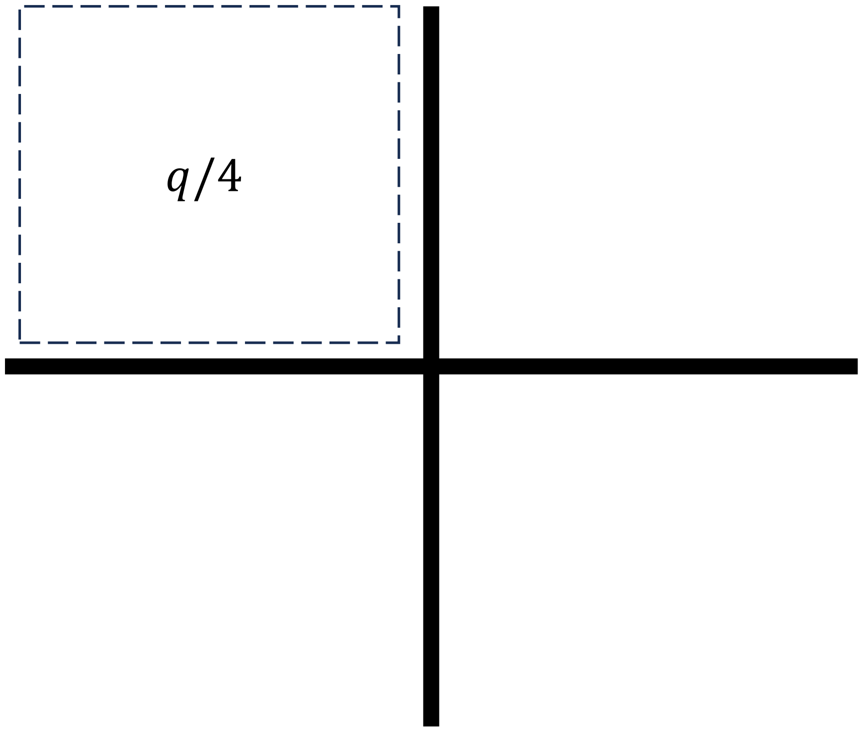


  - $\left(\frac{1}{2}\right)^{\mathrm{d_F}}$ =1/3, $d_F = 1.5849$ …<2
- The plain square
  - $\left(\frac{1}{2}\right)^{\mathrm{d}} = 1/4$, d=2



Fractal dimension

- $\left(\frac{1}{2}\right)^{d_F} = q/4, d_F = \frac{\log \frac{4}{q}}{\log 2} > 2$

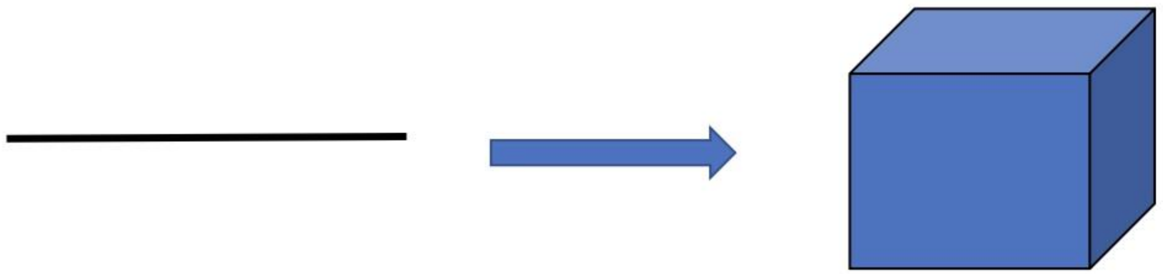


- Contrary to fractal objects, the hyperfractal dimension is larger than the euclidian dimension

- When $q \to 1, d_F \to 2$: the point process tends to uniform Poisson

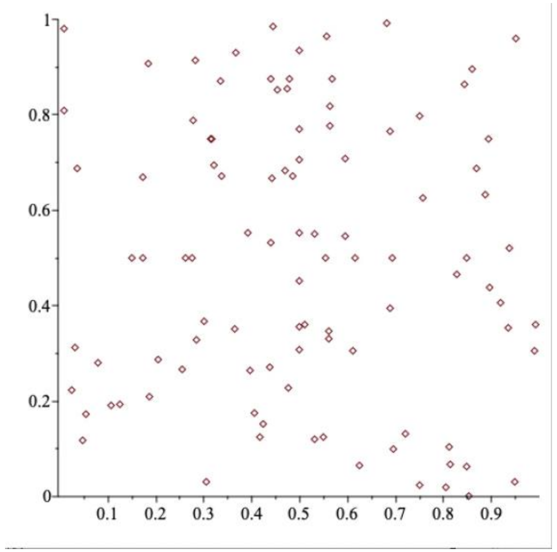
Hyperfractal measure

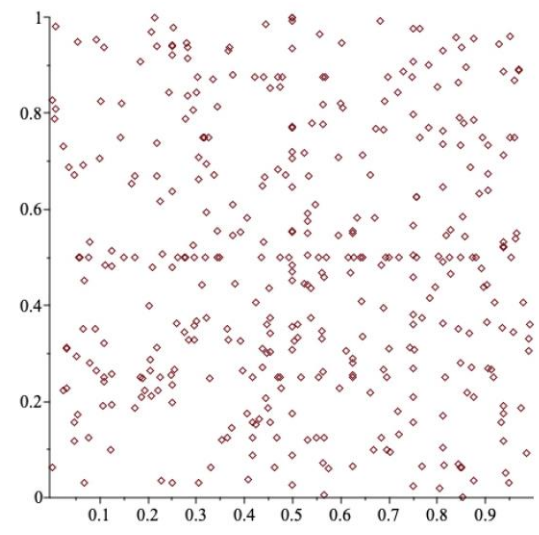
- Density measure are insensitive to euclidian dimension

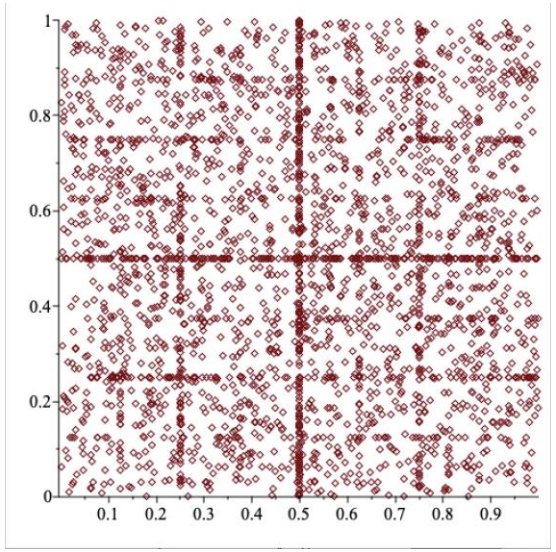


$$x = 0.10011101001111010010 0\ldots \quad \rightarrow \quad \begin{cases} y_1 = 0.1100111\ldots \\ y_2 = 0.0111100\ldots \\ y_3 = 0.0101000\ldots \end{cases}$$

- n moving cars: density decreases when depth increases $\lambda_k = n\left(\frac{p}{2}\right)\left(\frac{q}{2}\right)^k$







General Hyperfractal cities and traffic

City	d_F
Adelaide	2.8
Minneapolis	2.9
Nyon	2.3
Seattle	2.3

Obtained via the Zipf distribution of street densities:

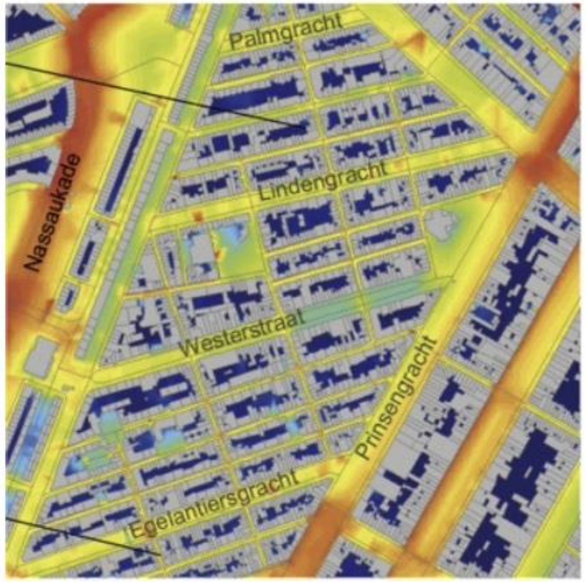
Density in pure hyperfractal: Street density=

Log(Street density)=(1-dF)*log(Distance covered by denser streets)

Jacquet, Philippe, and Dalia Popescu. "Self-similar geometry for ad-hoc wireless networks: Hyperfractals." *Geometric Science of Information: Third International Conference, GSI 2017, Paris, France, November 7-9, 2017*.

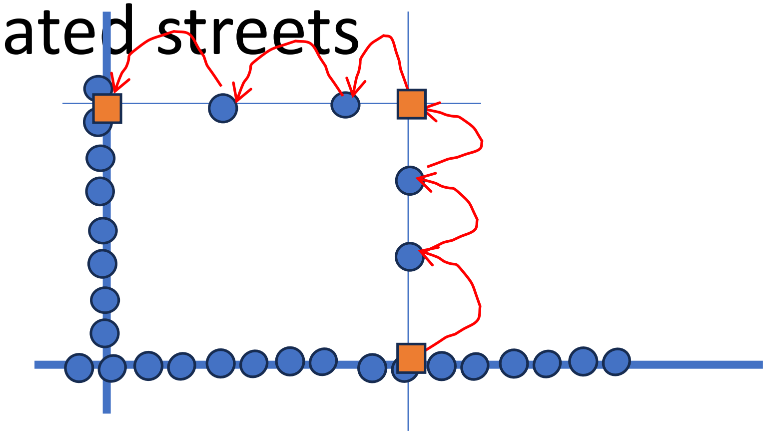
Delay tolerant network, information teleportation

- Urban wireless networking.
  - Canyon effect: radio propagates along streets but don’t penetrate buildings
  - Homothetic argument: For the same radio activity, range is proportional to inter-vehicle gap



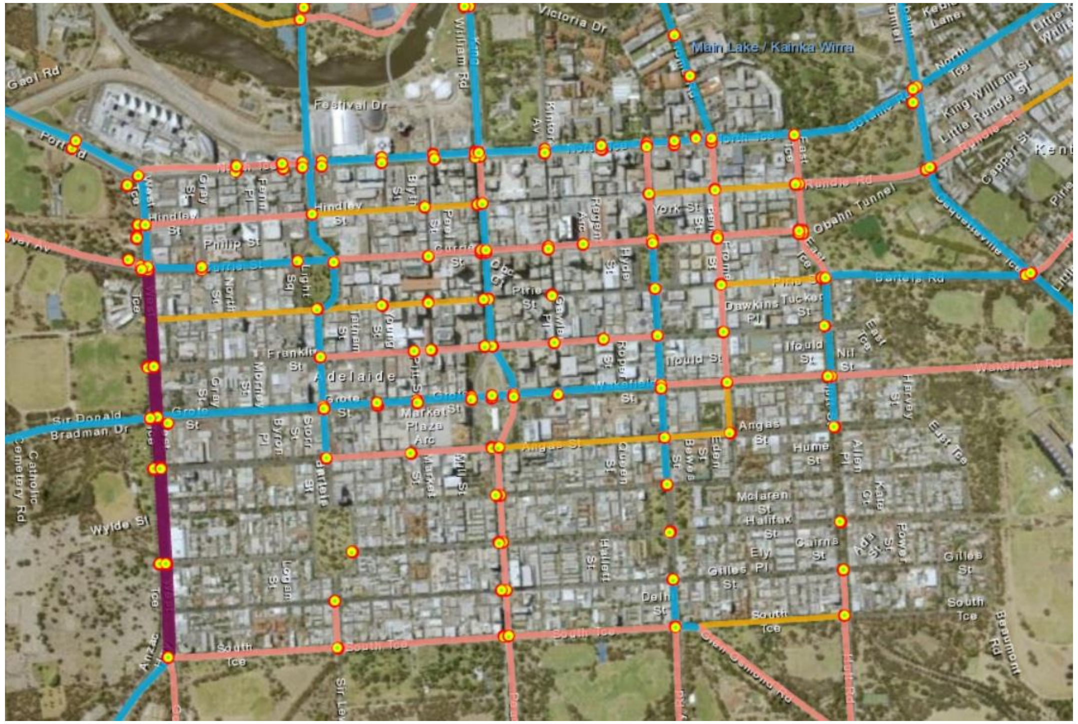
Multi-hop routing

- Short cuts via low densely populated streets

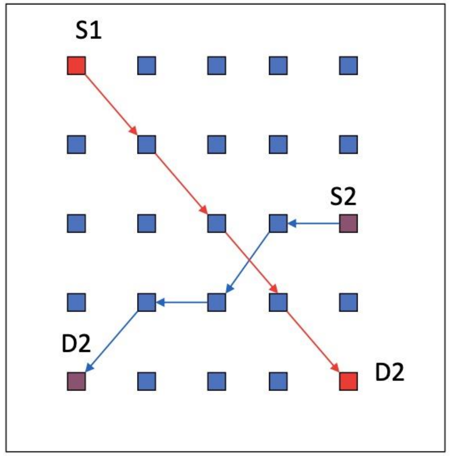


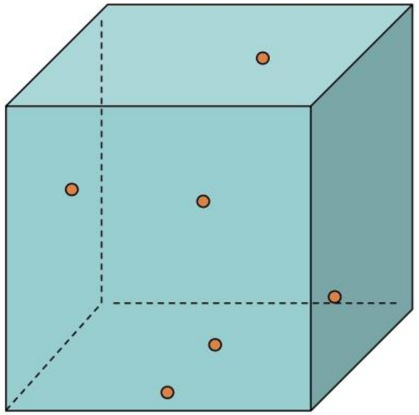
- Need of relays at intersection

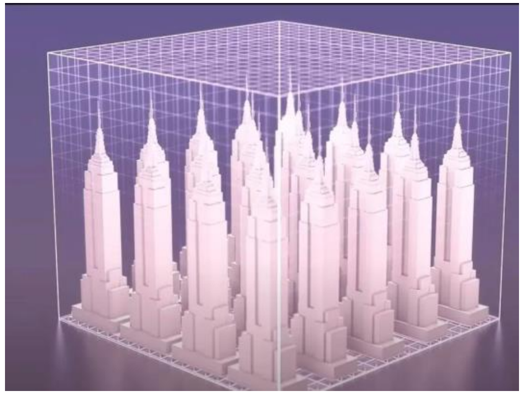
- Hyperfractal distribution of relays
  - Hyperfractal dimension d_r
  - Transport capacity
    - $T = n^{\frac{2}{\left(1+\frac{1}{d_F}\right)d_r}}$
  - When $d_r \to 2$
    - $T \to n^{1-\frac{1}{d_F+1}}$



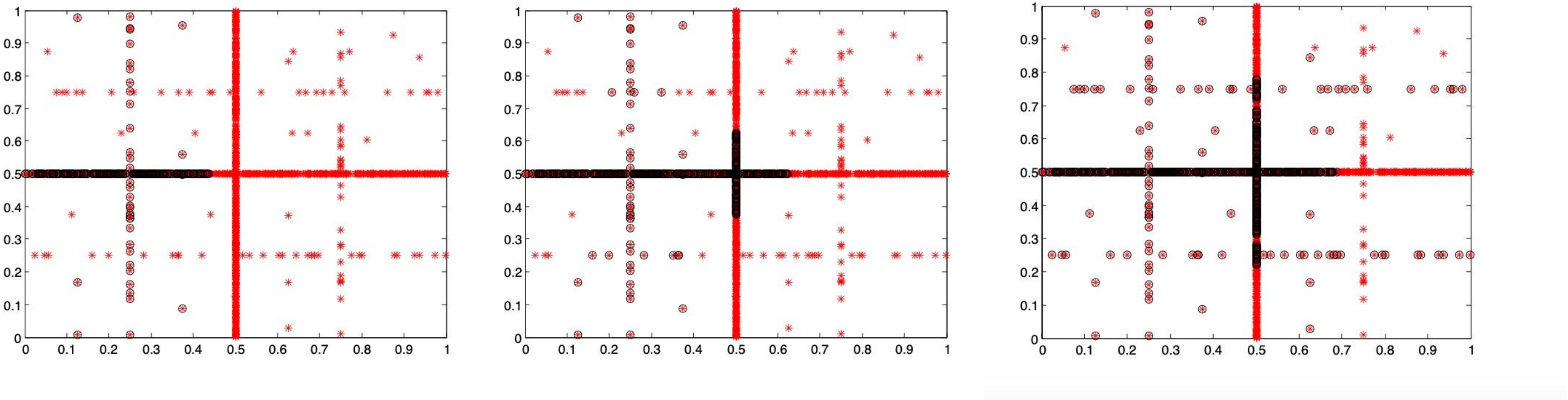
- Gupta-Kumar general result on multihop wireless network embedded in an euclidian space:
  - $T \sim n^{1-1/D}$
- An hyperfractal city is at least equivalent to a cube.
  - $T \to n^{1-\frac{1}{d_F+1}}$







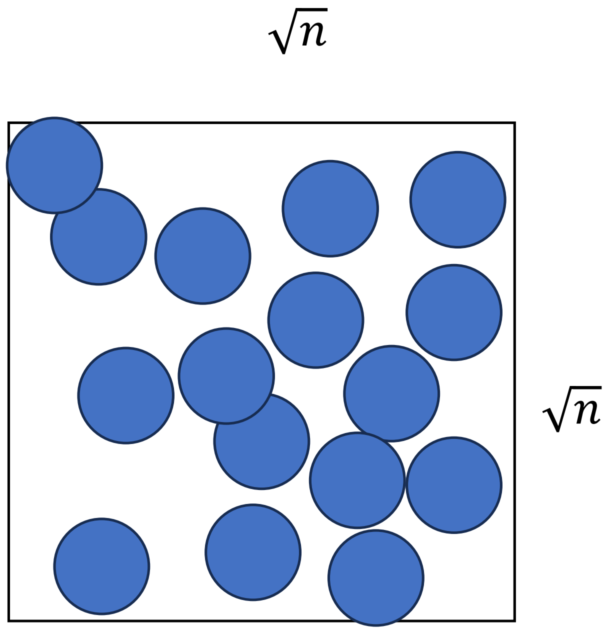
Information “teleportation”



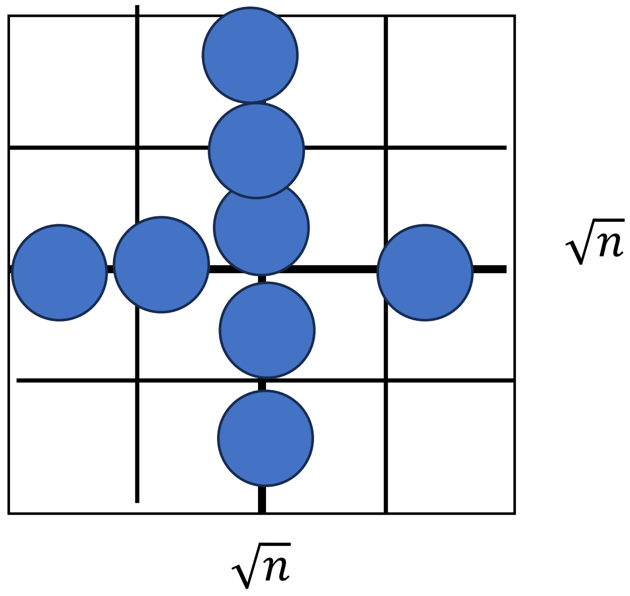
Jacquet, Philippe, Dalia Popescu, and Bernard Mans.
"Information dissemination in vehicular networks in an urban hyperfractal topology." *network* 6 (2018): 8.

Bounded radio range covering

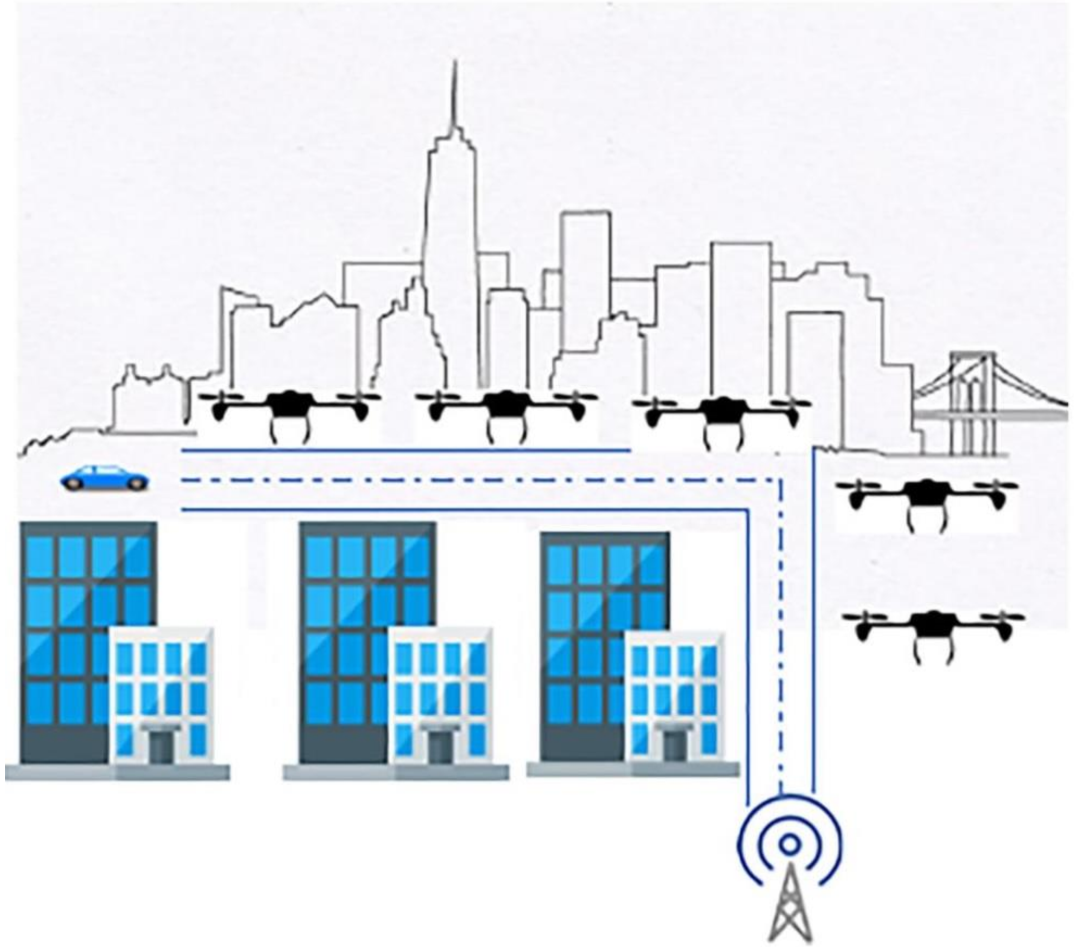
- Critical number of hot spots
  Uniform density $\mathrm{L_n} = \sqrt{n}$
  - In uniform, non fractal, $\Omega(n)$ hot spots are needed to cover 100%



  - In hyperfractal, only $\Omega(\sqrt{n})$ hot spots are needed to cover up to 100%



Using UAV for completing the covering

- 

Jacquet, Philippe, Dalia Popescu, and Bernard Mans.
"Connecting flying backhauls of drones to enhance vehicular networks with fixed 5g nr infrastructure."
IEEE INFOCOM 2020-IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS). IEEE, 2020.

Parking problem for car sharing

- Car sharing (Christine Fricker, Alessia Rigonat).
- Main difficulty: parking the shared car after the ride

Parking problem for car sharing

Busy street

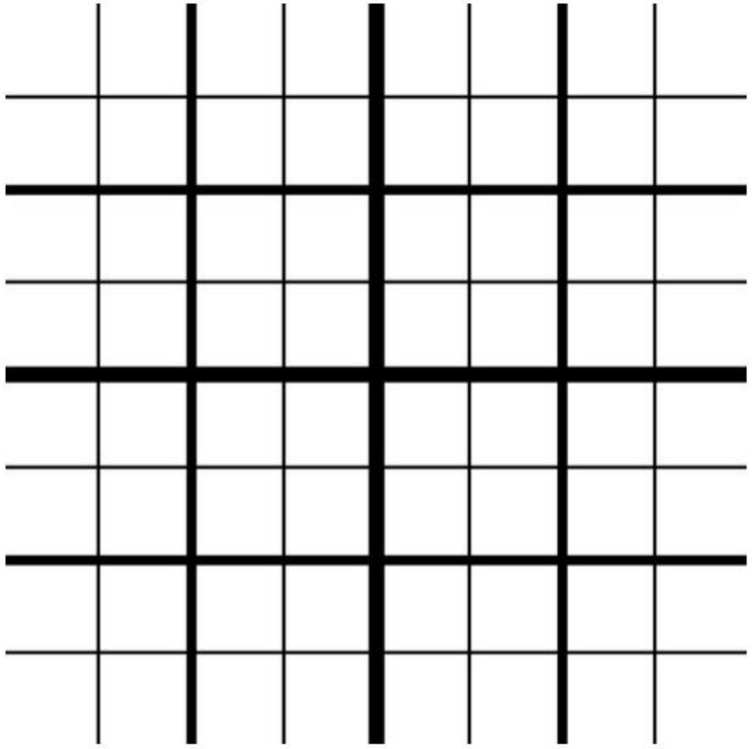
Residential street

- Only difference: frequency of parking turn over.

Hyperfractal parking turnover

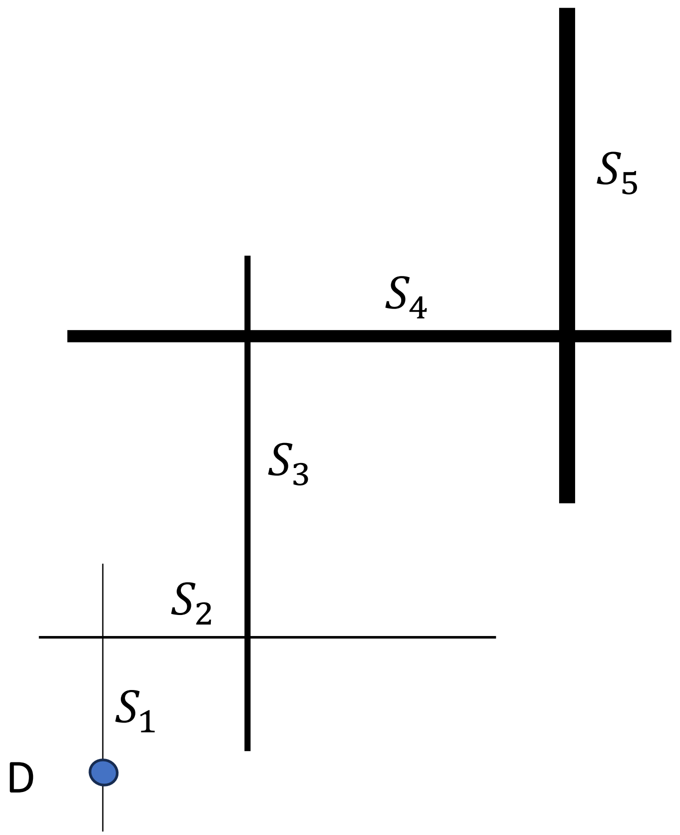
- Thick streets are with high turnover
  - Busy streets
- Thin streets are with the low turnover
  - Residential streets

- λ is the global city parking turnover rate



Quasi random walk for finding a free parking slot

- Primary car destination D is generally in a residential street
- From this spot: always turn into a more busy street
  - until finding a free parking slot



Distance traveled to parking

- $D(S_1, \dots, S_k) = \sum_i \frac{1-\exp(-\lambda_i |S_i|)}{\lambda_i} \prod_{j<i} \exp(-\lambda_j |S_j|)$
- $E[D] = \sum_{k_1 < \cdots < k_i < \cdots} \sum_i \frac{1}{2^{k_i}} \frac{1}{\left(1+\lambda\left(\frac{p}{2}\right)\left(\frac{q}{2}\right)^{k_i}\right)^2} \prod_{j>i} \frac{1}{1+\lambda\left(\frac{p}{2}\right)\left(\frac{q}{2}\right)^{k_j}}$

  - Asymptotically when $\lambda \to \infty$: $E[D] \sim \lambda^{-1/d_F}$
  - The distance to parking cannot be smaller than the order $\lambda^{-1/2}$

Generation of imaginary cities

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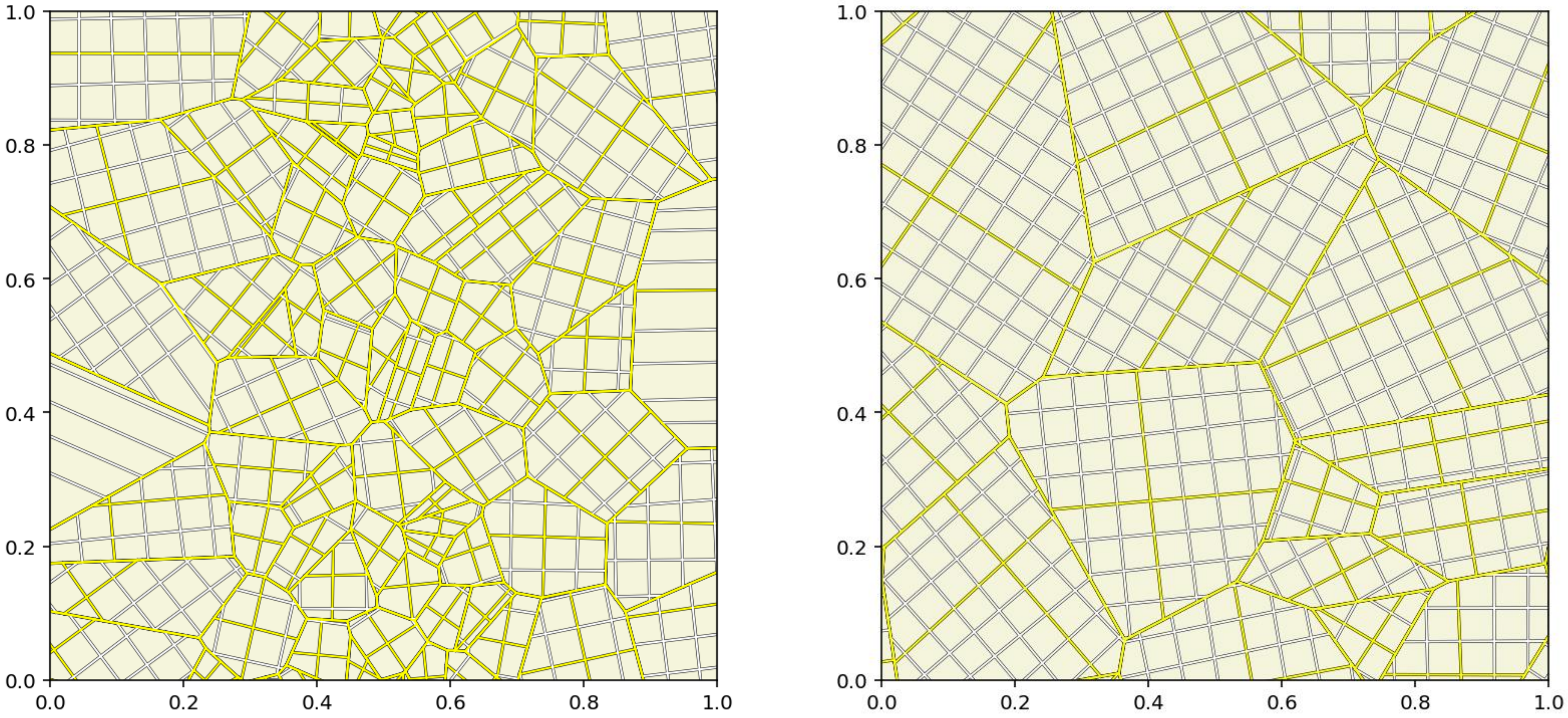

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